

Confidence Intervals, Hypothesis testing and p-values

Instructor's Notes

Purpose of the applet

The applet aims to make explicit the correspondence between: the inference based on a range of plausible values for the population parameter - confidence intervals; the inference based on the consistency of a sample statistic with a population parameter - a formal hypothesis test; and, the concept of the p -value.

About the applet

Essential features

The applet initially shows a 95% confidence interval, centred on a sample mean, \bar{x} , with the hypothesized mean, μ_0 , chosen to coincide with \bar{x} .

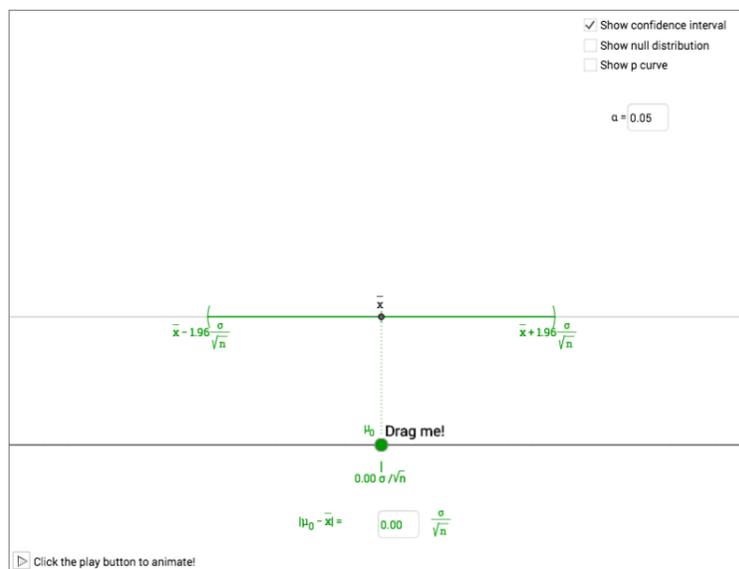


Figure 1: Starting position

This initial view can be changed by varying the modifiable features of the applet. These are:

- $\alpha = 0.05$ - specifies the level of confidence $100 - \alpha$ and can be changed by typing an alternate value in the input box;
- $|\mu_0 - \bar{x}| = 0.00 \frac{\sigma}{\sqrt{n}}$ - defines the distance between μ_0 and \bar{x} . Just like α , we can change it by typing in a value in the input box;
- μ_0 - indicates the location of the hypothesized mean and can be changed by dragging μ_0 to the right or to the left. Notice that this action automatically changes the value in the input box $|\mu_0 - \bar{x}|$;
- Show confidence interval - displays or hides the confidence interval, by checking the box;
- Show null distribution - displays or hides the curve of the pdf of null distribution;
- Show p curve - displays the p -curve which is associated the p -value;
- Show p-value - displays or hides the value of the p -value associated to the hypothesis test when either the "Show null distribution" or "Show p curve" check-boxes have been engaged. By engaging the check-box the applet displays the p -value in three distinct locations:
 - on the p -curve
 - above the curve of the pdf of the null distribution
 - and in the $p =$ input box. This input box also allows us to change the value of the p -value by typing in an alternate value.

Notation and colour coding

The colour coding of the applet reflects the intuitive associations of green with acceptance and red with rejection. Using Figure 2 as a reference, setting $\alpha = 0.05$ and starting from top to bottom, the green sections correspond to:

- the region where $\alpha < p \leq 1$
- the confidence interval region; and,

- the region of non-rejection of H_0 , $|\bar{x} - \mu_0| < 1.96\sigma/\sqrt{n}$

The red sections correspond to:

- the region where $p \leq \alpha$; and,
- the region of rejection of H_0 , $|\bar{x} - \mu_0| \geq 1.96\sigma/\sqrt{n}$.

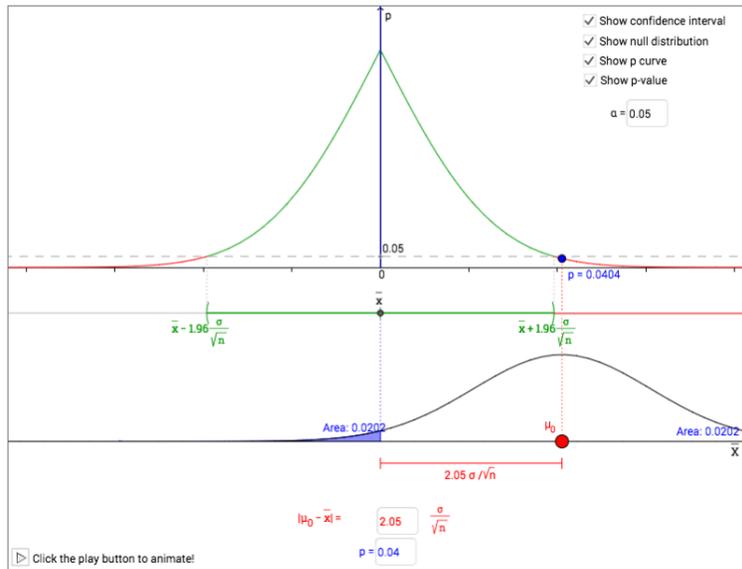


Figure 2: Distribution of \bar{X} under H_0 with the curve of the p -value

An example on how to use the applet

The applet can be used when introducing the concepts individually. However, the main aim of the applet is to illustrate the connection between a confidence interval, a formal hypothesis test and the p -value that may arise from the hypothesis test. The example that follows shows how this can be achieved by interlinking the three concepts. We assume that the applet is being used in the setting of a lecture and that the students have seen and understood beforehand the three concepts individually and are ready to see how the three concepts combine.

We use the the standard 95% confidence interval level and set μ_0 to coincide with \bar{x} , as displayed in Figure 1.

1. Starting with the applet as in Figure 1, the instructor asks the students:

How could we use the confidence interval for μ to test the null hypothesis that $\mu = \mu_0$, and in this case, if we were to do so, what would we conclude? Would we reject or not reject the null based on our confidence interval?

2. After that the instructor asks the students

What would we conclude if we varied the position of μ_0 to a position where, for example:

- 2.1. $|\mu_0 - \bar{x}| < 1.7$ (note that any position such that $0 < |\mu_0 - \bar{x}| < 1.96$ can be used in this case); and
- 2.2. $|\mu_0 - \bar{x}| = 2.5$ (note that any position such that $|\mu_0 - \bar{x}| \geq 1.96$ can be used in this case)

For each of the cases, after the students have answered the question, the instructor then uses the applet to confirm their answer. In 2.2 it is important to bring to their attention that the graph now shows μ_0 and the distance $|\mu_0 - \bar{x}|$ in red, indicating that we would reject the null, as displayed in Figure 3.

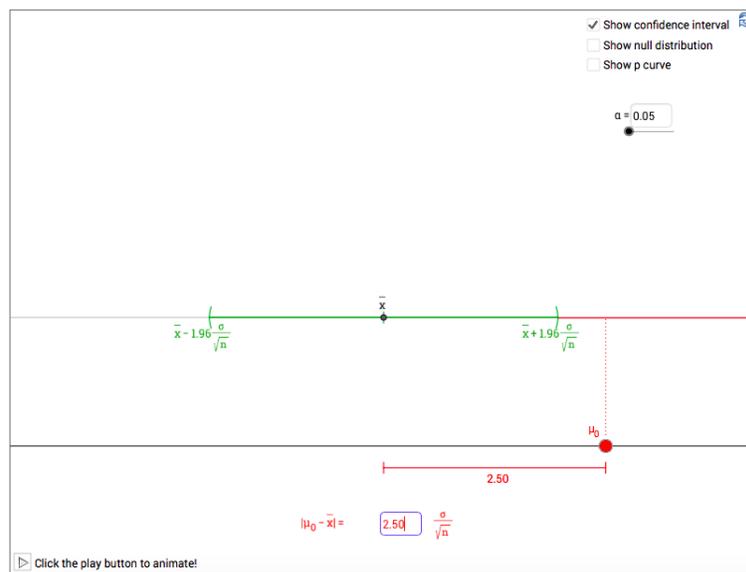


Figure 3: Reject H_0 based on the confidence interval

3. Now, leaving the applet in the position of Figure 3, the instructor can start to ask questions about the null distribution. For example,

- Is the distribution of H_0 going to be centered around μ_0 or around \bar{x} ?
- Does most of the distribution fall outside of the confidence interval or inside of the confidence interval?

and as before, the instructor then uses the applet to confirm the students' answers, by checking the box "show null distribution" to obtain a figure similar to that of Figure 4:

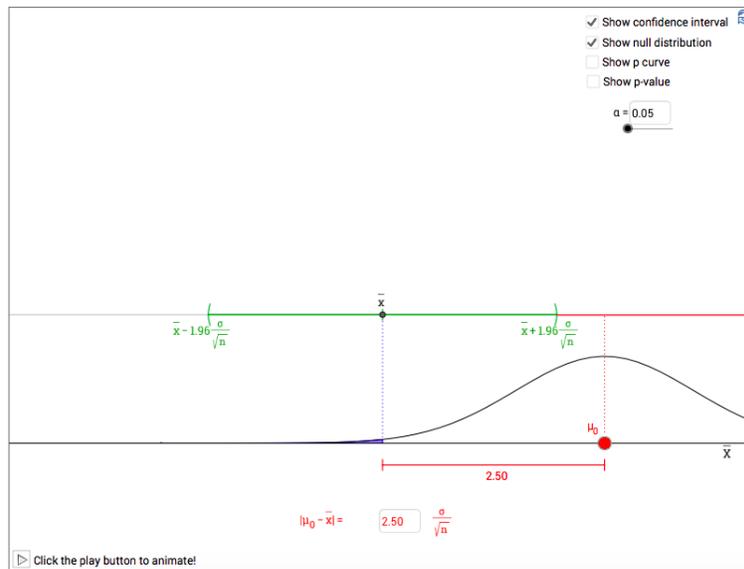


Figure 4: Reject H_0 based on the confidence interval and null distribution

4. Similarly, the instructor can now change the position of μ_0 to that of 2.1 and repeat the same questions as in 3.
5. Once 3 and 4 have been achieved, the instructor can repeat a similar exercise but now focusing on the p -value. Some of these questions can be, for example:
 - Is the p -value in 2.1 going to be greater, less than or equal to 0.05?
 - If we move μ_0 closer to \bar{x} , is the p -value going to increase or decrease?
 - If our standardized observed test-statistic is 1.5 will the p -value be less than or greater than 0.05. Note that for the students to answer this question they should consider the distance $|\mu_0 - \bar{x}| = 1.5\sigma/\sqrt{n}$.

As before, the instructor then uses the applet to confirm the students' answers, by engaging the "show p-value" button, which then shows, in the case of 2.2, what is displayed in Figure 5.

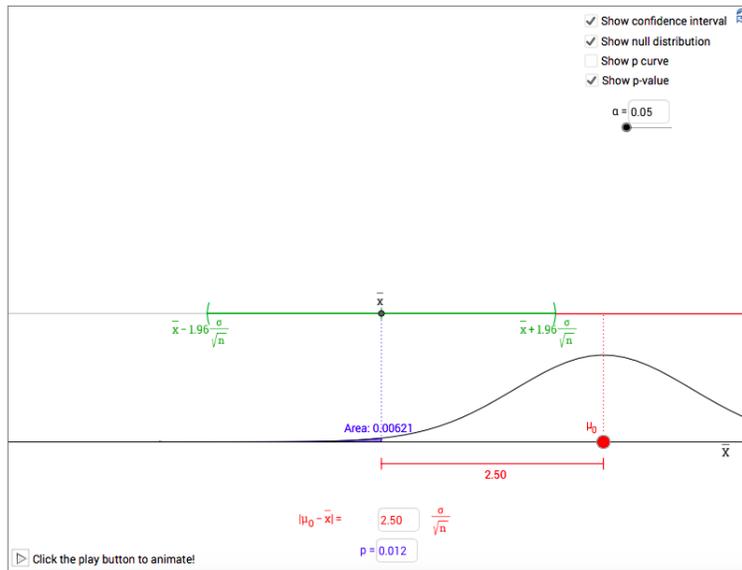


Figure 5: Reject H_0 based on the confidence interval with p -value

Notice now that by engaging the "show p-value" button, the value of p appears in the p -value input box on the bottom centre of the applet and in addition the value "Area=0.00621" appears on the graph of the null distribution. The instructor might want to ask the students, in this case, if the area that is coloured in blue is 0.00621 why is the p -value equal to 0.012. Once the students have answered, the instructor can then shift back to the position of 2.1 to show that the area is coloured on both sides of the distribution, corresponding to the definition of the p -value in a two-sided hypothesis test.

- In addition, or alternatively, the instructor can now engage the "show p curve" button to show or ask students about how the p -value changes when the position of μ_0 changes along the \bar{X} axis. By doing so, the p -value is displayed along the axis $\mu_0 - \bar{x}/\sigma/\sqrt{n}$, as displayed in Figure 6.

Note: The example shows one method of how the applet may be used to tie the three concepts together. However, the instructor might want to do a similar analysis but in the

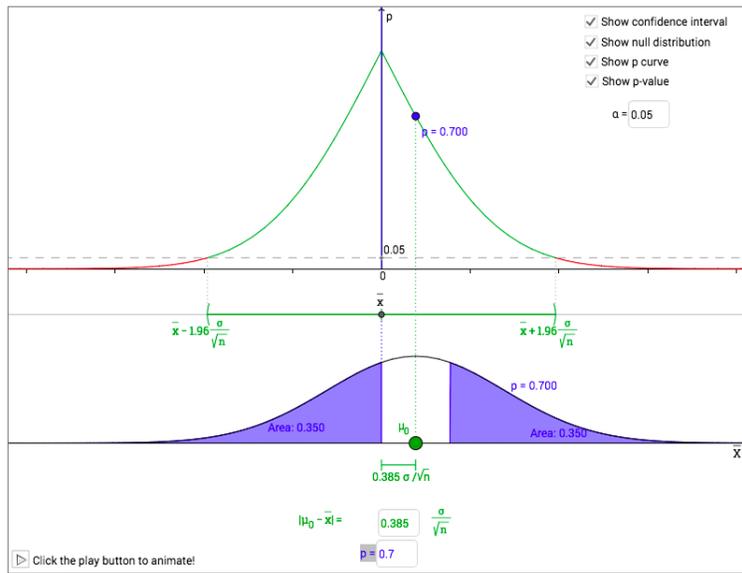


Figure 6: Applet with the p curve engaged

reverse order, for example, start off with the null distribution and then move over to the confidence interval or start off with the p curve to aid the understanding of the position of the null distribution with respect to \bar{x} . This can be easily achieved with the applet as it allows for different combinations of displays through its check boxes.

Extension/Discussion Questions

- How could you use the p -curve to create a confidence interval?
- Can a p -value of 1 be achieved?
- What does a 0% confidence interval look like?
- What does a 0% correspond to in a hypothesis test setting?